

Application of Multitaper Analysis to Wireless Communications Problems

(Invited Paper)

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Abstract—The paper presents two novel applications of Thomson Multitaper Analysis. It is shown how a wideband simulator of a double mobile MIMO channel could be developed based on geometrical channel model. It is also shown how modification of Discrete Prolate Spheroidal Sequences could be used to better estimation of sparse channels. A number of other potential applications is also mentioned.

I. INTRODUCTION

The goal of this paper is to discuss some applications of the Thomson Multitaper Analysis [1], [2] to problems encountered in communications [3], [4]. Only two applications are considered in details in this paper: flexible simulator of MIMO wideband channels (Section III) and channel estimation using modulated DPSS frames (Section IV). However, numerous other applications could be found. In particular we would like to emphasize application to Cognitive Radio problems as outlined in [5],[6].

Sum of Sinusoids (SoS) or Sum of Cisoids (SoC) simulators are popular ways of building channel simulators both in SISO and MIMO case [7], [8]. However, this approach is not very good option when prediction and estimation are simulated since it represents a signal as a sum of coherent components with large prediction horizon [9]. In this communication we develop an approach which allows to avoid this difficulty. The idea of a simulator combines representation of the scattering environment advocated in [8], [10] - [12] and the approach for a single cluster environment used in [13] - [16] with some important modifications.

The problem of estimation and prediction of a moderately fast fading Rayleigh/Rice channel is important in modern communications. The Wiener filter provides the optimum solution when the channel statistical characteristics are known [17]. However, in real-life applications basis expansions such as Fourier bases and discrete prolate spheroidal sequences (DPSS) have been adopted for such problems [18]-[16]. If the bases and the channel under investigation occupy the same band, accurate and sparse representations of channels are usually obtained [18]. However, a larger number of bases is required to approximate the channel with the same accuracy when the bandwidth of the basis function is mismatched and

larger than that of the signal. A need clearly exists for some type of overcomplete, redundant bases which accounts for a variety of scenarios. A recently proposed overcomplete set of bases called Modulated Discrete Prolate Spheroidal Sequences (MDPSS) [20] resolves the aforementioned issues. The bases within the frame are obtained by modulation and variation of the bandwidth of DPSSs in such a way as to reflect various scattering scenarios.

II. THEORETICAL BACKGROUND

The technique used here was introduced first in [2] and further discussed in [1]. We adopt notations used in the original paper [2]. Let $x(n)$ be a finite duration segment of a stationary process, $n = 0, \dots, N - 1$. It can be represented as

$$x(n) = \int_{-1/2}^{1/2} \exp(j2\pi f[n - (N - 1)/2]) dZ(f) \quad (1)$$

according to the Cramer theorem [9]. It is emphasized in [2] that goal of the spectral analysis is to estimate moments of $Z(f)$, in particular its first and second moments from a finite sample $x(n)$. However, spectral analysis is often reduced only to considering the second centered moment

$$S(f)df = \mathcal{E} \{ |dZ(f)|^2 \} \quad (2)$$

well known as the spectrum (power spectral density). In the case of continuous spectrum the first moment of $dZ(f)$ is zero and it is usually not considered. However, in the case of a line or mixed spectrum one obtains

$$\mathcal{E} \{ dZ(f) \} = \sum_k \mu_k \delta(f - f_k) df \quad (3)$$

where μ_k is the amplitude of the harmonic with frequency f_k . Knowledge of the discrete and continuous part of the spectrum allows to solve a number of applied problems in wireless communications such velocity estimation, K -factor estimation, prediction, *etc.*

The Discrete Prolate Spheroidal Sequences (DPSS) are defined as solutions to the Toeplitz matrix eigenvalue problem

[2], [21]

$$\lambda_k(N, W)u_k(N, W; n) = \sum_{m=0}^{N-1} \frac{\sin(2W(n-m))}{\pi(n-m)} u_k(N, W; m) \quad (4)$$

Their discrete Fourier transform (DFT) is known as Discrete Prolate Spheroidal Wave Functions (DPSWF) [21]

$$U_k(N, W; f) = \sum_{n=0}^{N-1} u_k(N, W; n) \exp(-j2\pi n f) \quad (5)$$

In particular, if $f = 0$, equation (5) can be rewritten as

$$U_k(N, W; 0) = U_k(0) = \sum_{n=0}^{N-1} u_k(N, W; n) \quad (6)$$

A number of spectral estimates, called eigen coefficients could be obtained using DPSS $u_k(N, W; n)$ as time-domain windows

$$Y_k(f) = \sum_{n=1}^{N-1} x(n)u_k(N, W; n) \exp(-j2\pi n f) \quad (7)$$

Since for a single narrow line spectral component at $f = f_0$, $2f_0 > W$

$$\mathcal{E}\{Y_k(f)\} = \mu U_k(N, W; f - f_0) + \mu^* U_k(N, W; f + f_0) \quad (8)$$

complex magnitude μ could be estimated by minimizing error between the eigen spectrum $Y_k(f)$ and $\mu(f)U_k(N, W; 0)$. The result of such minimization is a simple linear regression [9] of $Y_k(f)$ on $U_k(N, W; 0)$ and is given by

$$\hat{\mu}(f) = \frac{1}{\sum_{k=0}^{K-1} |U_k(0)|^2} \sum_{k=0}^{K-1} U_k^*(0) Y_k(f) \quad (9)$$

The fact that the estimate $\hat{\mu}(f)$ is the linear regression allows to use standard regression F -test [22] for significance of the line component with amplitude $\hat{\mu}(f)$ at frequency f . This could be achieved by examining the term [2]

$$F(f) = (K-1) \frac{|\hat{\mu}(f)|^2}{\epsilon^2(\hat{\mu}, f)} \sum_{k=0}^{K-1} |U_k^*(0)|^2 \quad (10)$$

The location of the local maximum of $F(f)$ provides an estimate of the line component of the spectrum. The hypothesis that there is a line component with magnitude $\hat{\mu}(f_0)$ at $f = f_0$ is accepted if $F(f)$ has maximum at $f = f_0$ and

$$F(f_0) > F_\alpha(K) \quad (11)$$

where $F_\alpha(K)$ is the threshold for significance level α and $K-1$ degrees of freedom. Values of $F_\alpha(K)$ can be found in standard books on statistics [22].

Estimation of spectrum in the vicinity of a line spectrum component at $f = f_0$ is given by

$$\hat{S}(f) = \frac{1}{K} \sum_{k=0}^{K-1} |Y_k(f) - \hat{\mu} V_k(f - f_0)|^2 \quad (12)$$

and

$$\hat{S}(f) = \frac{1}{K} \sum_{k=0}^{K-1} |Y_k(f)|^2 \quad (13)$$

otherwise.

It is recommended that the original sequence $x(n)$ is zero-padded to create a mesh of length 4–10 times longer than the original N . This is essential to avoid missing a line spectrum component which is far from the grid frequency [23].

III. SUM OF FINITE CLUSTERS SIMULATOR

A. Geometry of the problem

Let us first consider a single cluster scattering environment, shown in Fig. 1. It is assumed that both sides of the link are equipped with multielement uniform linear array antennas and both are mobile. The transmit array has N_T isotropic elements separated by distance d_T while the receiver side has N_R elements separated by distance d_R . Both antennas are assumed to be in the horizontal plane; however extension on the general case is straightforward. The antennas are moving with velocities v_T and v_R respectively such that the angle between corresponding broadside vectors and the velocity vectors are α_T and α_R respectively. Furthermore, it is assumed that the impulse response is sampled at the rate F_{st} and channel is sounded with the rate F_s impulse responses per second. The carrier frequency is f_0 . Practical values can be found in standards [8], [11].

The space between the antennas consist of a single scattering cluster whose center is seen at the the azimuth ϕ_{0T} and co-elevation θ_T from the receiver side and the azimuth ϕ_{0R} and co-elevation θ_R . The angular spread in the azimuthal plane is $\Delta\phi_T$ on the receive side and $\Delta\phi_R$ on the transmit side. No spread is assumed in the co-elevation dimension to simplify calculations and due to low array sensitivity to the co-elevation spread. We also assume that $\theta_R = \theta_T = \pi/2$ to shorten equations. Corresponding corrections are rather trivial. The angular spread on both sides is assumed to be small comparing to the angular resolution of the arrays due to a large distance between the antennas and the scatterer:

$$\Delta\phi_T \ll \frac{2\pi\lambda}{(N_T-1)d_T}, \quad \Delta\phi_R \ll \frac{2\pi\lambda}{(N_R-1)d_R} \quad (14)$$

The cluster, due to a finite angular extent, produces certain delay spread $\Delta\tau$ variation of the impulse response. This spread is assumed to be relatively small, not exceeding a few sampling intervals $T_s = 1/F_{st}$.

B. Single Cluster Simulator

It is well known that the angular spread (dispersion) in the impulse response leads to spatial selectivity [24] which could be described by corresponding spatial covariance function

$$\rho(d) = \int_{-\pi}^{\pi} \exp\left(j2\pi \frac{d}{\lambda} \phi\right) p(\phi) d\phi \quad (15)$$

where $p(\phi)$ is distribution of the AoA or AoD. Since the angular size of clusters are assumed to be much smaller than

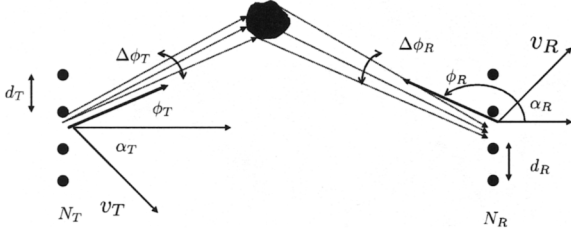


Fig. 1. Geometry of a single cluster problem.

the antennas angular resolution, one can further assume the following simplifications: a) the distribution of AoA/AoD are uniform and b) joint distribution $p_2(\phi_T, \phi_R)$ of AoA/AoD is

$$p_2(\phi_T, \phi_R) = p_{\phi_T}(\phi_T)p_{\phi_R}(\phi_R) = 1/\Delta\phi_T\Delta\phi_R \quad (16)$$

It was shown in [25] that corresponding spatial covariance functions are modulated sinc functions

$$\rho(d) \approx \exp\left(j\frac{2\pi d}{\lambda}\sin\phi_0\right)\text{sinc}\left(\Delta\phi\frac{d}{\lambda}\cos\phi_0\right) \quad (17)$$

The correlation function of the form (17) gives rise to a correlation matrix between antenna elements which can be decomposed in terms of frequency modulated Discrete Prolate Spheroidal Sequences (MDPSS) [26], [21], [20]:

$$\mathbf{R} \approx \mathbf{W}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H\mathbf{W}^H = \sum_{k=0}^D \lambda_k \mathbf{u}_k \mathbf{u}_k^H \quad (18)$$

where $\mathbf{\Lambda} \approx \mathbf{I}_D$ is the diagonal matrix of size $D \times D$ [21], \mathbf{U} is $N \times D$ matrix of the DPSS and $\mathbf{W} = \text{diag}\{\exp(j2\pi d/\lambda \sin nd_A)\}$. Here λd_A is distance between the antenna elements, N number of antennas, $1 \leq n \leq N$ and $D \approx \lceil 2\Delta\phi\frac{d}{\lambda}\cos\phi_0 \rceil + 1$ is the effective number of degrees of freedom generated by the process with the given covariance matrix \mathbf{R} . For a narrow spread clusters number of degrees of freedom is much less than the number of antennas $D \ll N$ [21]. Thus, it could be inferred from equation (18) that the desired channel impulse response $\mathbf{H}(\omega, \tau)$ could be represented as a double sum or tensor product

$$\mathbf{H}(\omega, t) = \sum_{n_t=1}^{D_T} \sum_{n_r=1}^{D_R} \sqrt{\lambda_{n_t}\lambda_{n_r}} \mathbf{u}_{n_t}^{(r)} \mathbf{u}_{n_t}^{(t)H} h_{n_t, n_r}(\omega, t) \quad (19)$$

In the extreme case of a very narrow angular spread on both sides, $D_R = D_T = 1$ and $\mathbf{u}_1^{(r)}$ and $\mathbf{u}_1^{(t)}$ are well approximated by the Kaiser windows [2]. The channel corresponding to a single scatterer is of course a rank one channel given by

$$\mathbf{H}(\omega, t) = \mathbf{u}_1^{(r)} \mathbf{u}_{n_t}^{(t)H} h_{n_t, n_r}(\omega, t) \quad (20)$$

considering the shape of the functions $\mathbf{u}_1^{(r)}$ and $\mathbf{u}_1^{(t)}$ one can conclude that in this scenario angular spread is achieved by modulating the amplitude of the spatial response of the channel on both sides. It is also worth noting that representation (19) is the Karhunen-Loeve series [17] in spatial domain and therefore produces smallest number of terms needed to represent the process selectivity in spatial domain. It is also easy to see

that such modulation becomes important only when number of antennas is significant.

Similar results could be obtained in frequency and Doppler domains. Indeed, let the mean excess delay τ is associate with the cluster under consideration and $\Delta\tau$ is the delay spread in time of arrival of signals from this cluster. In addition let it be desired to provide a proper representation of the process in the bandwidth $[-W : W]$ using N_F equally spaced samples. Assuming that the variation of power is relatively minor within $\Delta\tau$ delay window, we once again recognize that variation of channel in frequency domain can be described as a sum of modulated DPSS of length N_F and the time bandwidth product $W\Delta\tau$. The number of MDPSS needed for such representation is approximately $D_F = 2W\Delta\tau + 1$ [21]:

$$\mathbf{h}(\omega, t) = \sum_{n_f=1}^{D_f} h_{n_f}(t) \mathbf{u}_{n_f}^{(\omega)} \quad (21)$$

Finally, in the Doppler domain, the mean resulting Doppler frequency could be calculated as

$$f_D = \frac{f_0}{c} [v_T \cos(\phi_{T0} - \alpha_T) + v_R \cos(\phi_{R0} - \alpha_R)] \quad (22)$$

The angular extent of the cluster from sides causes the Doppler spectrum to widen by the following amount

$$\Delta f_D = \frac{f_0}{c} [v_T \Delta\phi_T v_T |\sin(\phi_{T0} - \alpha_T)| + v_R \Delta\phi_R |\sin(\phi_{R0} - \alpha_R)|] \quad (23)$$

Once again, due to a small angular extent of the cluster it could be assumed that the widening of the Doppler spectrum is relatively narrow and no variation within the Doppler spectrum is of importance. Therefore, if it is desired to simulate the channel on the interval of time $[0 : T_{max}]$ then this could be accomplished by summing $D = 2\Delta f_D T_{max} + 1$ MDPS:

$$\mathbf{h}_d = \sum_{n_d=0}^D \xi_{n_d} \sqrt{\lambda_{n_d}} \mathbf{u}_{n_d}^{(d)} \quad (24)$$

where ξ_{n_d} are independent zero mean complex Gaussian random variable of unit variance.

Finally, the derived representation could be summarized in tensor notions as follows. Let $\mathbf{u}_{n_t}^{(t)}$, $\mathbf{u}_{n_r}^{(r)}$, $\mathbf{u}_{n_f}^{(\omega)}$ and $\mathbf{u}_{n_d}^{(d)}$ are DPSS corresponding to the transmit, receive, frequency and Doppler time dimensions of the signal with the "domain-dual domain" products given by $\Delta\phi_T \frac{d}{\lambda} \cos\phi_{T0}$, $\Delta\phi_R \frac{d}{\lambda} \cos\phi_{R0}$, $W\Delta\tau$ and $T_{max}\Delta f_D$ respectively. Then a sample of a MIMO frequency selective channel with corresponding characteristics could be generated as

$$\mathcal{H}_4 = \mathcal{W}_4 \circledast \sum_{n_t}^{D_T} \sum_{n_r}^{D_R} \sum_{n_f}^{D_F} \sum_{n_d}^d \sqrt{\lambda_{n_t}^{(t)} \lambda_{n_r}^{(r)} \lambda_{n_f}^{(\omega)} \lambda_{n_d}^{(d)}} \xi_{n_t, n_r, n_f, n_d} \mathbf{1} \mathbf{u}_{n_t}^{(r)} \times \mathbf{2} \mathbf{u}_{n_r}^{(r)} \times \mathbf{3} \mathbf{u}_{n_f}^{(\omega)} \times \mathbf{4} \mathbf{u}_{n_d}^{(d)} \quad (25)$$

where \mathcal{W}_4 is a tensor composed of modulating sinusoids

$$\mathcal{W}_4 = \mathbf{1} \mathbf{w}^{(r)} \mathbf{2} \mathbf{w}^{(t)} \mathbf{1} \mathbf{w}^{(\omega)} \mathbf{1} \mathbf{w}^{(d)} \quad (26)$$

$$\begin{aligned}
\mathbf{w}^{(r)} &= \left[1; \exp\left(j2\pi\frac{d_R}{\lambda}\right); \cdots; \exp\left(j2\pi\frac{d_R}{\lambda}(N_R - 1)\right) \right] \\
\mathbf{w}^{(t)} &= \left[1; \exp\left(j2\pi\frac{d_T}{\lambda}\right); \cdots; \exp\left(j2\pi\frac{d_T}{\lambda}(N_T - 1)\right) \right] \\
\mathbf{w}^{(\omega)} &= [1; \exp(j2\pi\Delta F\tau); \cdots; \exp(j2\pi\Delta F(N_F - 1))] \\
\mathbf{w}^{(d)} &= [1; \exp(j2\pi\Delta f_D T_s); \cdots; \exp(j2\pi\Delta f_D(T_{max} - T_s))]
\end{aligned}$$

and \odot is the Hadamard product of two tensors [27].

C. Multi-Cluster environment

The generalization of the model suggested above is straightforward. The channel between the transmitter and the receiver is represented as a set of clusters, each described as in Section (III-B). The total impulse response is superposition of independently generated impulse response tensors from each cluster

$$\mathcal{H}_4 = \sum_{k=0}^{N_c-1} \sqrt{P_k} \mathcal{H}_4(k), \quad \sum_{k=1}^{N_c} P_k = P \quad (27)$$

where N_c is total number of clusters, $\mathcal{H}_4(k)$ is a normalized response from the k -th cluster $\|\mathcal{H}_4(k)\|_F^2 = 1$ and $P_k \geq 0$ represents power of k -th cluster and P is the total power.

It is important to mention here that such representation does not need to correspond to a physical cluster distribution. It rather reflect interplay between signals radiated, arriving from certain direction with a certain excess delay, ignoring particular mechanism of propagation. Therefore it is possible, for example, to have two clusters with the same AoA and AoD but different excess delay. Alternatively, it is possible to have two clusters which correspond to the same AoD and excess delay but very different AoA.

Equations (25) and (27) reveal connection between SoC approach [8] and the suggested algorithms: one can consider (25) as a modulated cisoid. Therefore, the simulator suggested above could be considered as a Sum of Modulated Cisoids simulator.

In addition to space dispersive components, the channel impulse response may contain a number of highly coherent components, which can be modelled as pure complex exponents. Such components described either direct LoS path or specularly reflected rays with very small phase diffusion in time. Therefore equation (27) should be modified to account for such components:

$$\begin{aligned}
\mathcal{H}_4 &= \sqrt{\frac{1}{1+K}} \sum_{k=0}^{N_c-1} \sqrt{P_{ck}} \mathcal{H}_4(k) + \\
&\quad \sqrt{\frac{K}{1+K}} \sum_{k=0}^{N_s-1} \sqrt{P_{sk}} \mathcal{W}_4(k) \quad (28)
\end{aligned}$$

Here N_s is a number of specular components including LoS and K is generalized Rice factor describing ratio between powers of specular P_{sk} and non-coherent/diffusive components P_{ck}

$$K = \frac{\sum_{k=0}^{N_s-1} P_{sk}}{\sum_{k=0}^{N_c-1} P_{ck}} \quad (29)$$

While distribution of the diffusive component is Gaussian by construction, the distribution of the specular component may not be Gaussian. A more detailed analysis is beyond the scope of this manuscript and will be considered elsewhere. We also leave a question of identifying and distinguishing coherent and non-coherent components to a separate manuscript.

IV. MDPSS FRAMES FOR CHANNEL ESTIMATION AND PREDICTION

When both the DPSS and the channel under investigation occupy the same frequency band, then usually DPSS provide accurate and sparse representations [18]. However, problems arise when the channel is centered around some frequency $|\nu_o| > 0$ and the occupied bandwidth is smaller than $2W$. In such situations, a larger number of DPSS is required to approximate the channel with the same accuracy despite the fact that such narrowband channel is more predictable than a wider band channel [9]. In order to find a better basis so-called Modulated Discrete Prolate Spheroidal Sequences (MDPSS) have been proposed in [20]:

$$M_k(N, W, \omega_m; n) = \exp(j\omega_m n) u_k(N, W; n), \quad (30)$$

where $\omega_m = 2\pi\nu_m$ is the modulating frequency. Using the proposed bases, one can form frames [28], precomputed in such a way as to reflect various scattering scenarios. Therefore, representation in the overcomplete basis can be made sparse due to the richness of such a basis. Since the expansion into simple bases is not unique, a fast, convenient and unique projection algorithm cannot be used. Fortunately, efficient algorithms, known generically as pursuits [29], [30] can be used to achieve a sparse representation. For full details on MDPSS and its implementation please refer to [20].

A. Numerical Properties of MDPSS

The performance of the MDPSS estimator has been compared with the Slepian basis expansion DPSS approach [18] for a certain radio environment. Fig. 2 depicts the results of the estimation accuracy for a WSSUS channel with a uniform power angle profile (PAS) with central AoA $\phi_0 = 5$ degrees and spread $\Delta = 20$ degrees [20]. It is clear that the mean square error for MDPSS estimator is several orders of magnitude lower than the value for the Slepian basis expansion estimator based on DPSS.

Additional numerical studies considering various angles of arrivals and spreading angles have been conducted [20]. The results clearly indicated that the MDPSS frames are a more accurate estimation tool for the assumed channel model. For the considered angles of arrival and spreading angles, the MDPSS estimator consistently provided lower MSE in comparison to the Slepian basis expansion estimator based on DPSS. The advantage of the MDPSS stems from the fact that these bases are able to describe different scattering scenarios.

V. CONCLUSION

Multitaper Analysis is very powerful tool for a number of communication problems which are based on estimation of

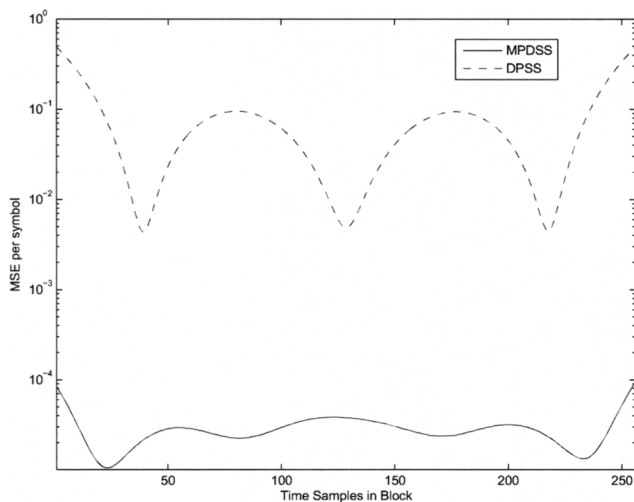


Fig. 2. Mean square error per symbol for MDPSS (solid) and DPSS (dashed) mobile channel estimators for the noise-free case.

the power spectral density or its accurate simulation. As an example we developed a wideband simulator of mobile-to-mobile fading channel which generalized conventional sum of sinusoids simulators. Such an approach resolves a problem of overly optimistic simulations of systems with prediction. It also allows for simulation of radio-scene changing environment and non-stationary environments. We also have shown that use of frames based on Thomson tapers allows significantly reduce number of bases functions needed for signal reconstruction in the case of non-isotropic scattering. A number of other applications, including generalized factor estimation, prediction of channel with mixed spectrum are being prepared for publication.

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